Q1

1

State whether the following mappings are one-to-one, many-to-one, one-to-many or many-to-many.

(i) $f: x \mapsto \tan x$

(ii) $f: x \mapsto |\frac{1}{x}|$

(iii) $f: x \mapsto \sqrt{x^2}$

(iv) $f: x \mapsto \pm\sqrt{25 - x^2}$

[4]

i) MANY TO ONE

$\tan 60 = \sqrt{3}$

$\tan 240 = \sqrt{3}$

ii) MANY TO ONE

$\left|\frac{1}{2}\right| = \left|\frac{1}{-2}\right|$

iii) MANY TO ONE

$\sqrt{2^2} = 2$

$\sqrt{-2^2} = 2$

$\sqrt{}$ ROOT WILL ALWAYS GIVE NON NEGATIVE VALUE

iv) MANY TO MANY

$\pm\sqrt{25 - 4^2} = \pm 3$

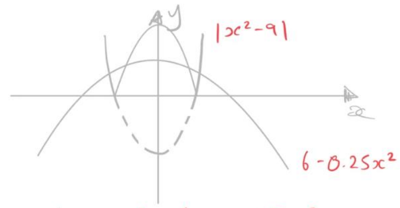
$\pm\sqrt{25 - (-4)^2} = \pm 3$

Q2

2

Solve the equation $|x^2 - 9| = 6 - 0.25x^2$, giving your answers in exact form.

[3]



$|x^2 - 9| = 6 - 0.25x^2$

$x^2 - 9 = 6 - 0.25x$

$1.25x^2 = 15$

$x^2 = 12$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

$x^2 - 9 = -6 + 0.25x^2$

$0.75x^2 = 3$

$x^2 = 4$

$x = \pm 2$

$\boxed{x = \pm 2\sqrt{3} \qquad x = \pm 2}$

Q3a

3a

The functions $f(x)$, $g(x)$ are defined as follows

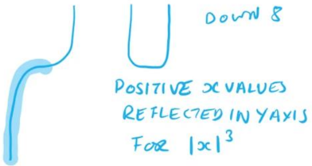
$$f(x) = |x^3 - 8| \qquad x \in \mathbb{R}$$
$$g(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = fg(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

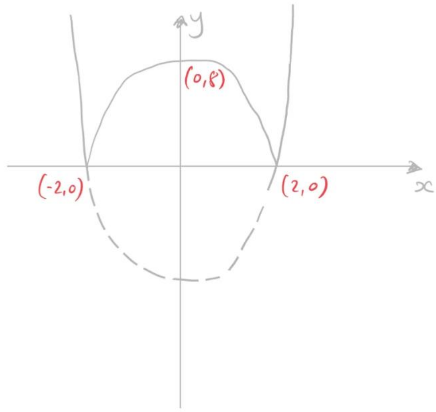
[3]

(b) There are between 0 and 4 solutions to the equation $fg(x) = c$, where c is a real number. Determine the values of c that produce each number of solutions.

[3]



a) $fg(x) = \left||x|^3 - 8\right|$



Q3b

3b

The functions $f(x)$, $g(x)$ are defined as follows

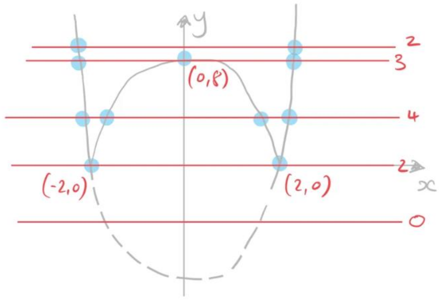
$$f(x) = |x^3 - 8| \qquad x \in \mathbb{R}$$
$$g(x) = |x| \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = fg(x)$, stating the coordinates of all points where the graph intercepts the coordinate axes.

[3]

(b) There are between 0 and 4 solutions to the equation $fg(x) = c$, where c is a real number. Determine the values of c that produce each number of solutions.

[3]



b)

$c < 0$ NO SOLUTIONS

$c = 0$ OR $c > 8$ TWO SOLUTIONS

$c = 8$ THREE SOLUTIONS

$0 < c < 8$ FOUR SOLUTIONS

NO VALUE OF c FOR ONE SOLUTION

Q4a

4a

(a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{g}(x)|$ where

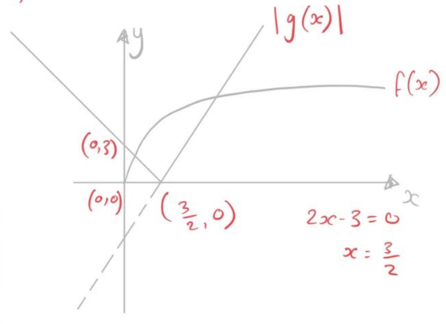
$\mathrm{f}(x) = \sqrt{x}$ $\quad x \geq 0$
$\mathrm{g}(x) = 2x - 3$ $\quad x \in \mathbb{R}$

Label the points at which the graphs intersect the coordinate axes. [3]

(b) Solve the equation $\mathrm{f}(x) = |\mathrm{g}(x)|$. [3]

(c) Which of the solutions to $\mathrm{f}(x) = |\mathrm{g}(x)|$ is ***not*** a solution to $\mathrm{f}(x) = \mathrm{g}(x)$? [1]

a)



$2x - 3 = 0$

$x = \frac{3}{2}$

Q4b

4b

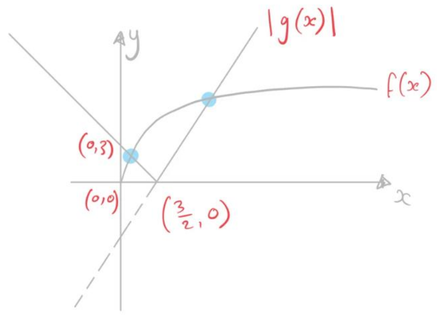
(a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{g}(x)|$ where

$\mathrm{f}(x) = \sqrt{x}$ $\quad x \geq 0$
$\mathrm{g}(x) = 2x - 3$ $\quad x \in \mathbb{R}$

Label the points at which the graphs intersect the coordinate axes. [3]

(b) Solve the equation $\mathrm{f}(x) = |\mathrm{g}(x)|$. [3]

(c) Which of the solutions to $\mathrm{f}(x) = |\mathrm{g}(x)|$ is ***not*** a solution to $\mathrm{f}(x) = \mathrm{g}(x)$? [1]



save my exams

b) SOLVE MOD EQUATION BY SQUARING BOTH SIDES

$\sqrt{x} = |2x - 3|$

SQUARE

$x = (2x - 3)^2$

$x = 4x^2 - 12x + 9$

$4x^2 - 13x + 9 = 0$

$(4x - 9)(x - 1)$

$\boxed{x = \frac{9}{4} \qquad x = 1}$

Q4c

4c

(a) On the same axes, sketch the graphs of $y = \mathrm{f}(x)$ and $y = |\mathrm{g}(x)|$ where

$\mathrm{f}(x) = \sqrt{x}$ $\quad x \geq 0$
$\mathrm{g}(x) = 2x - 3$ $\quad x \in \mathbb{R}$

Label the points at which the graphs intersect the coordinate axes.

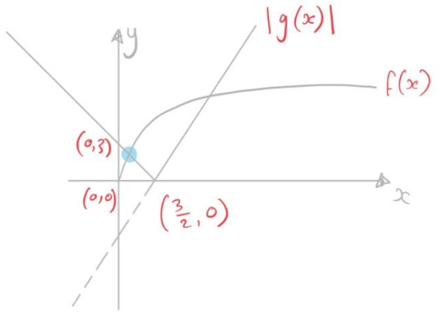
[3]

(b) Solve the equation $\mathrm{f}(x) = |\mathrm{g}(x)|$.

[3]

(c) Which of the solutions to $\mathrm{f}(x) = |\mathrm{g}(x)|$ is ***not*** a solution to $\mathrm{f}(x) = \mathrm{g}(x)$?

[1]



c) $x = 1$ $\quad x = \frac{9}{4}$

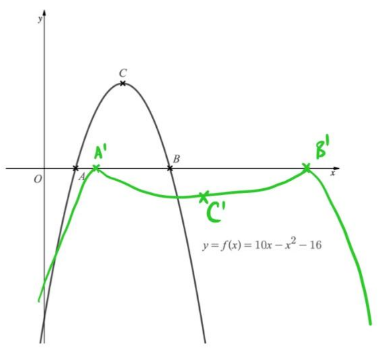
$f(x) = g(x)$

$\boxed{x = 1 \text{ IS NOT A SOLUTION}}$

Q5a

5a

A sketch of the graph with equation $y = f(x)$ where $f(x) = 10x - x^2 - 16$ is shown below.
Points A and B are the x-axis intercepts and point C is the maximum point on the graph.



(a) On the diagram above, sketch the graph of $y = -\left|\frac{1}{4}\mathrm{f}\left(\frac{1}{2}x\right)\right|$ labelling the image of the points A, B and C with A', B' and C'.

[3]

(b) Show that the area of ABC is twice the area of triangle $A'B'C'$.

[4]

a) HORIZONTAL STRETCH SF 2
VERTICAL STRETCH SF $\frac{1}{4}$
MOD REFLECTION ABOVE x AXIS
VERTICAL REFLECTION IN x AXIS

USE $f(x)$ FUNCTION TO CALCULATE ABC

$A = (2, 0)$ $\quad B = (8, 0)$ $\quad C = (5, 9)$

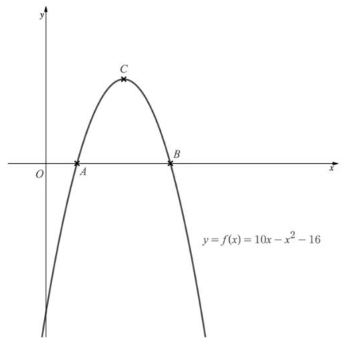
TRANSFORM CO-ORDINATES TO FIND A'B'C'

$A' = (4, 0)$ $\quad B' = (16, 0)$ $\quad C' = (10, -2.25)$

Q5b

5b

A sketch of the graph with equation $y = f(x)$ where $f(x) = 10x - x^2 - 16$ is shown below.
Points A and B are the x-axis intercepts and point C is the maximum point on the graph.



(a) On the diagram above, sketch the graph of $y = -\left|\frac{1}{4}f\left(\frac{1}{2}x\right)\right|$ labelling the image of the points A, B and C with A', B' and C'.

[3]

(b) Show that the area of ABC is twice the area of triangle $A'B'C'$.

[4]

b) USING COORDINATES FOR ABC AND A'B'C'

$A = (2, 0)$ $\quad B = (8, 0)$ $\quad C = (5, 9)$

$A' = (4, 0)$ $\quad B' = (16, 0)$ $\quad C' = (10, -2.25)$

AREA $\frac{1}{2}bh$

ABC $\quad \frac{1}{2}(8-2) \times 9 = 27$

A'B'C' $\quad \frac{1}{2}(16-4) \times 2.25 = 13.5$

$\boxed{ABC = 27 \text{ units}^2 \quad A'B'C' = 13.5 \text{ units}^2 \quad ABC = 2(A'B'C')}$

Q6

6

The function $f(x) = e^{3x} - x - 6$ is transformed by a sequence of transformations as described below.

1. Horizontal stretch by scale factor 3,
2. The modulus of the function is then taken,
3. Reflection in the y-axis.

Write down the resulting transformation in terms of $f(x)$ as well as an expression in terms of x.

[4]

$f(x) = e^{3x} - x - 6$

1. x MULTIPLIED BY $\frac{1}{3}$

$f\left(\frac{1}{3}x\right) = e^{\cancel{3}\left(\frac{1}{\cancel{3}}x\right)} - \frac{1}{3}x - 6$

$= e^{x} - \frac{1}{3}x - 6$

2. MODULUS

$\left|f\left(\frac{1}{3}x\right)\right| = \left|e^{x} - \frac{1}{3}x - 6\right|$

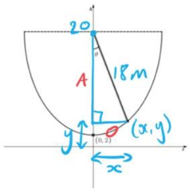
3. HORIZONTAL REFLECTION

$\boxed{\left|f\left(-\frac{1}{3}x\right)\right| = \left|e^{-x} + \frac{1}{3}x - 6\right|}$

Q7a

7a

A swing boat fairground ride is modelled as moving forwards and backwards along the path of a semi-circle, radius 18 m, as shown in the diagram below.



(a) Show that, for $0 \le \theta \le \frac{\pi}{2}$,

(i) the x-coordinate of the boat is given by $x = 18\sin\theta$,

(ii) the y-coordinate is given by $y = 20 - 18\cos\theta$.

[3]

The model is refined so that the coordinates of the boat can be calculated from the time, t seconds, after the boat is set in motion. The x and y coordinates are now given by

$$x = 18\sin Bt \qquad y = 20 - 18|\cos Bt|$$

where B is a constant.

(b) (i) Briefly explain why the modulus of $\cos\theta$ is required for the y- coordinate.

(ii) Given that the time between the boat reaching its maximum height at either end of the ride is 8 seconds, find the value of B.

[3]

(c) For $0 \le t \le 4$, find the times when the boat is equidistant from the ground and horizontally from the origin.

[3]

a)

USING SOH CAH TOA

i) $O = \sin\theta \times H = x$

$\boxed{x = 18\sin\theta}$

ii) $A = \cos\theta \times H = 20 - y$

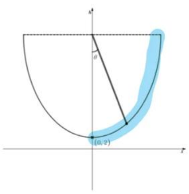
$y = 20 - \cos\theta \times H$

$\boxed{y = 20 - 18\cos\theta}$

Q7b

7b

A swing boat fairground ride is modelled as moving forwards and backwards along the path of a semi-circle, radius 18 m, as shown in the diagram below.



(a) Show that, for $0 \le \theta \le \frac{\pi}{2}$,

(i) the x-coordinate of the boat is given by $x = 18\sin\theta$,

(ii) the y-coordinate is given by $y = 20 - 18\cos\theta$.

[3]

The model is refined so that the coordinates of the boat can be calculated from the time, t seconds, after the boat is set in motion. The x and y coordinates are now given by

$$x = 18\sin Bt \qquad y = 20 - 18|\cos Bt|$$

where B is a constant.

(b) (i) Briefly explain why the modulus of $\cos\theta$ is required for the y- coordinate.

(ii) Given that the time between the boat reaching its maximum height at either end of the ride is 8 seconds, find the value of B.

[3]

(c) For $0 \le t \le 4$, find the times when the boat is equidistant from the ground and horizontally from the origin.

[3]

b) $-1 \le \cos\theta \le 1$

i) IF $\cos\theta$ IS NEGATIVE $-18\cos Bt$ WOULD BE POSITIVE

MODULUS IS NEEDED TO PREVENT Y-COORDINATE GOING ABOVE 20 WHICH IS THE MAX POSSIBLE HEIGHT

$0 \le \theta \le \frac{\pi}{2}$ COVERS HALF OF SEMI CIRCLE

ii) 8 SECONDS $= \pi$ $\quad 180° = \pi$

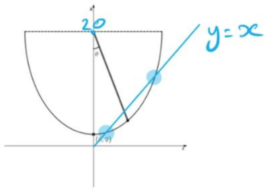
$8B = \pi$

$\boxed{B = \frac{\pi}{8}}$

Q7c

7c

A swing boat fairground ride is modelled as moving forwards and backwards along the path of a semi-circle, radius 18 m, as shown in the diagram below.



(a) Show that, for $0 \le \theta \le \frac{\pi}{2}$,

(i) the x-coordinate of the boat is given by $x = 18\sin\theta$,

(ii) the y-coordinate is given by $y = 20 - 18\cos\theta$.

[3]

The model is refined so that the coordinates of the boat can be calculated from the time, t seconds, after the boat is set in motion. The x and y coordinates are now given by

$$x = 18\sin Bt \qquad y = 20 - 18|\cos Bt|$$

where B is a constant.

(b) (i) Briefly explain why the modulus of $\cos\theta$ is required for the y- coordinate.

(ii) Given that the time between the boat reaching its maximum height at either end of the ride is 8 seconds, find the value of B.

[3]

(c) For $0 \le t \le 4$, find the times when the boat is equidistant from the ground and horizontally from the origin.

$x = y$

[3]

c) $y = x$ ON EQUATION OF CIRCLE

$(x-a)^2 + (y-b)^2 = r^2 \qquad (a, b) = (0, 20)$

$r = 18$

$x^2 + (y-20)^2 = 18^2$

$x^2 + y^2 - 40y + 400 = 324$

SUB $y = x$

$2x^2 - 40x + 76 = 0$

$x = 10 \pm \sqrt{62} \qquad x = 18\sin\left(\frac{\pi}{8}t\right)$

$\sin\left(\frac{\pi}{8}t\right) = \frac{10 \pm \sqrt{62}}{18}$

$\sin^{-1}$

$\frac{\pi}{8}t = 0.11838\ldots \qquad 1.452\ldots$

$t = 0.30147\ldots \qquad t = 3.6985\ldots$

$\boxed{t = 0.3 \text{ SECONDS AND } t = 3.7 \text{ SECONDS}}$